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The industrial production and application of historical building component heritage from the perspective of digital technology module chain

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K E Y W O R D S

ABSTRACT

digital technology module chain, construction of historical buildings, cultural relics protection, industrial production, education and communication The protection of historical architectural heritage is the key research object of the ancient architecture industry, and the research on the partial construction of buildings is gradually deepening.Nowadays, the use of digital means to research and protect architectural heritage has become an important way for many scholars.However, there are many types of digital software on the market with complex functions, and common digital means include X-spectrum, Jingdiao JDP, point cloud data, 3D printing, etc.Based on the simple construction of dougong, this paper analyzes the method of digital technology module chain, and applies it to the field of architectural construction such as murals and sculptures. This digital technology module chain can not only study and analyze the construction of historical buildings, but also meet the needs of cultural relics preservation, industrial production, education and communication.Finally, it is expected that through the promotion of this digital technology module chain, the protection of historical architectural heritage and cultural communication will be promoted in a more optimized direction.

1. Introduction

In the field of Chinese ancient architecture preservation, the application of digital technologies has become increasingly important, serving as a vital tool in promoting cultural heritage protection. The introduction of virtual reality (VR) technology offers a new perspective for cultural heritage conservation, providing immersive experiences that help the public gain a deeper understanding and appreciation of the cultural value of ancient buildings^[1]. Simultaneously, digital methods such as 3D modeling and data storage have significantly enhanced the efficiency and accuracy of architectural heritage preservation^[2]. As a cutting-edge tool, 3D printing technology demonstrates enor-

mous potential in the replication and restoration of historical artifacts, not only reducing damage to the originals but also improving the precision of repairs^[3].Furthermore, the integration of 3D scanning technology with 3D printing provides innovative solutions for cultural heritage preservation, making the restoration and protection of artifacts more effective^[4].Against this backdrop, China's relevant policies and legislative frameworks are continually evolving to promote the integration of digital applications in cultural heritage conservation, ensuring sustainability and social participation^[5].These studies highlight the significance of digital technologies in the preservation of ancient architecture and their impact on future conservation efforts.

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However, with the continuous emergence of digital software, the diversity, complexity, and lack of cohesion among digital tools make it impossible for any single technology to fulfill the combined demands of research, production, dissemination, and teaching. In this era of rapid digital software development, it is increasingly challenging for a single application to address the needs of the market, research, and education sectors. Amid the uncontrolled proliferation of digital technologies, a modular chain that organizes and connects these tools in an orderly fashion is notably absent. This paper aims to explore a novel digital module that is wellsuited for the protection and development of ancient architectural heritage. By advancing the digital transformation of heritage protection, this module is intended to better align with the market demand for industrial production, foster secondary innovation, and enhance communication and educational outreach.

Therefore, this paper primarily investigates the feasibility and sustainable development of the digital module chain through systematic analysis and empirical research, examining its potential for the preservation, production, and dissemination of ancient architectural heritage. By studying simplified dougong structures and promoting applications in murals, sculptures, and other handicrafts, this research demonstrates the value of the digital module chain, providing a modular approach to operation and management in an era of continuous digital advancement.

This paper examines the modular chain facilitating the transition from ancient architectural heritage preservation to industrial production, the modular chain connecting heritage preservation to subsequent research, and the modular chain linking industrial production to cultural dissemination. Additionally, the verification and adaptation of various models aim to enhance the relevance of historical heritage to social demand and supply.

2. Literature review

Over the past two decades, digital technologies such as 3D scanning, 3D printing, Building Information Modeling (BIM), and X-ray spectroscopy have dramatically transformed the field of cultural heritage preservation. These techniques offer detailed documentation and precise restoration methods that have redefined approaches to preserving historical structures and artifacts. This review synthesizes recent research on the application of these technologies in cultural heritage, highlighting both their advancements and limitations.

2.1. Application of point cloud data

3D digitization techniques, as examined in studies by Ioannides and Magnenat-Thalmann, have provided highly accurate models of cultural artifacts that can be stored digitally for future restoration efforts^[6]. The strength of these technologies lies in their ability to capture intricate details without physically interfering with the object. However, the high costs associated with equipment and the expertise required for 3D modeling present significant barriers^{[6][7]}. Future developments could focus on reducing costs and making the technology more user-friendly, potentially through the integration of open-source software^{[6][8]}.

Point cloud data has also been employed in creating digital Building Information Models (BIM) for historical buildings.Karasaka and Ulutas demonstrated how point cloud data enhances the accuracy of H-BIM, allowing for better analysis of structural integrity^[8].Despite its effectiveness, challenges arise in managing large datasets and ensuring the resolution of scans is sufficient for detailed modeling^[9].Research suggests that automating parts of the modeling process could reduce the time and expertise needed for such projects^{[8][9]}.

2.2. Application of 3D printing technology

The role of 3D printing in the restoration of architectural elements has grown, with several studies exploring its potential to replicate damaged components with a high degree of accuracy^[11].Prieto and Kumar's work highlights the ability of 3D printing to reproduce complex architectural features, though limitations remain concerning the durability of printed materials compared to original building materials^[11].Advances in material science could help develop more resilient and environmentally friendly 3D printing materials^[11].

In combination with 3D printing, X-ray spectroscopy has proven to be a valuable tool for noninvasive material analysis.X-ray spectroscopy enables conservators to identify the composition of materials in historical objects, aiding in the development of appropriate restoration methods^[13].However, the reliance on specialized equipment, which is often expensive and requires controlled environments, limits the widespread use of this technology in the field^{[13][14]}The development of portable spectrometers could broaden the use of this method in on-site conservation^[14].

2.3. Application of BIM technology

BIM, another crucial technology in heritage conservation, has been used to manage the long-term preservation of historical buildings.Fai and Graham argue that BIM facilitates more efficient documentation and helps coordinate conservation activities across large projects^[15].Despite its advantages, BIM requires a high level of skill to create accurate models, especially when dealing with irregular or deteriorated structures^{[15][16]}.Training programs for heritage professionals could address the skill gap and improve the adoption of BIM technologies in conservation^[16].

Recent studies have also emphasized the sustainability of using digital technologies, particularly 3D printing, in heritage conservation.3D printing has been shown to reduce material waste and allows conservators to replicate damaged elements without altering the original structures^[17].However, the energy demands of 3D printers are high, and their environmental benefits are currently offset by this significant energy consumption^{[17][18]}.Future research could focus on making 3D printing more energy-efficient, reducing its environmental impact while maintaining its conservation benefits^[17].

To sum up, advanced technologies such as X-ray spectrometer, point cloud data technology, BIM technology, Jingdiao software, and 3D printing technology play their own unique roles in the protection and utilization of cultural relics, providing multi-form technical support for the inheritance and development of cultural relics, and carrying out certain forms of prevention and control of possible risks in the future, such as data damage, man-made damage to cultural relics, and damage to natural factors.

3. Methods

3.1. Research Philosophy

Three-dimensional scanning technology is currently widely used in the protection of cultural relics, first using X-ray spectrometer to scan cultural relics, uploading data, and secondary processing through software such as point cloud data.X-spectroscopy technology has played an important role in the field of preservation and restoration of cultural relics, and the processing of 3D models is an indispensable link. The Meixian Museum has successfully modeled and restored some of the cultural relics in its collection by using 3D laser scanning technology, outlining a comprehensive and detailed outline for the cultural relics, realizing the all-round collection and collection of cultural relics information, the virtualization and digital construction of exhibitions, and the restoration of cultural relics ^[19]. Three-dimensional data achieves the whole process from collection to analysis, analysis to restoration, restoration to display.

To enable broader use and dissemination of cultural relics, three-dimensional data can be extended to support industrial production and cultural creativity. Taking the construction data of dougong as an example, this process begins with collecting data via X-ray spectrometry, refining the 3D printing master model, enhancing carving software designs, and ultimately producing the final mold. Each stage meets specific goals: X-ray spectrometry for preservation, design refinement for restoration, mold production for industrial-scale replication, and subsequent development for wider dissemination. This seamless process-from data archiving to design, from development to production-offers a new approach to preserving and sharing cultural relics through digital means.

In this era, the future trend driven by "digital +" has become very clear, and it is no longer just a simple "digital +" single industry.Now, "Digital+", a dynamic and innovative development strategy that integrates multiple industries, is rapidly emerging.The specific idea is shown in the figure 1.

3.2. Data module process optimization based on point cloud technology and 3D printing

3.2.1. Point cloud data algorithm processing

The collection and processing of point cloud data play a significant role in the protection and research of historical architectural heritage, typically gathered using laser scanners or 3D acquisition sensors. During data collection and analysis, specific algorithmic formulas are required. The data is then processed through algorithms and translated using BIM software, offering a novel approach to the preservation of historical architectural heritage.

ICP algorithm (Iterative Closest Point) is an important algorithm for point cloud registration. Specifically, it is as follows: Equation (1):

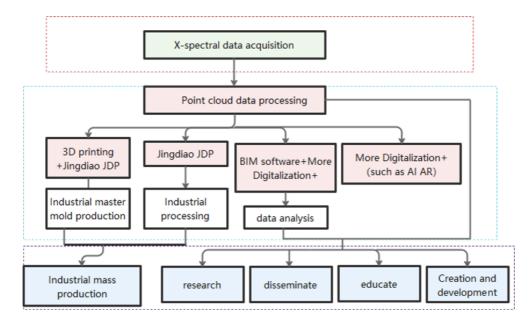


Fig. 1. Conceptual framework

$$R, t = \arg\min_{R, t} \sum_{i=1}^{N} \|Rp_i + t - q_i\|$$
(1)

In the equation, P_i represents the points collected in the source point cloud data, q_i represents the nearest neighboring points in the target point cloud, and R and t represent the rotation matrix and translation vector, respectively. By controlling the change in the calculated quantities, the minimum distance between the point clouds is found, aligning and verifying the source point cloud with the target point cloud.

After data processing, to better integrate with BIM software and establish a link, the point cloud data needs to be segmented.Based on RANSAC (Random Sample Consensus) model fitting, algorithmic analysis is conducted, as shown in the following formula (2):

In the equation, P_i represents the point data in the

$$d(p_i, M) = distance(p_i, M) \le \epsilon$$
 (2)

point cloud, M represents the geometric model, and ϵ epsilon ϵ represents the distance threshold.

The BIM + 3D scanning technology model is regarded as the core part of the initial data collection process.By collecting the point cloud data generated from the dougong (traditional Chinese architectural brackets), this data is then converted into LAS format and loaded into BIM software .This enables the linking and analysis of point data.

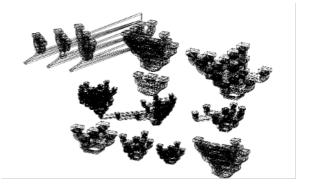


Fig. 1. BIM processing data model

3.2.2. Curve optimization and path processing

The Bezier curve defines a parametric curve through a set of control points, and is an algorithm in computer graphics, 3D modeling, and other fields ^[20]. The curve algorithm is used to control the generation of smooth curves, which restores the original form of the construction and provides a theoretical algorithm basis for restoration and preservation. The parametric equation is described in (3):

In the equation, B(t) represents the parametric form of the Bezier curve, P_i denotes the control

$$B(t) = \sum_{i=0}^{n} B_{i,n}(t)P_i$$
(3)

points, t is the parameter with a range between 0 and 1, and $B_{i,n}(t)$ is the basis function of the Bezier

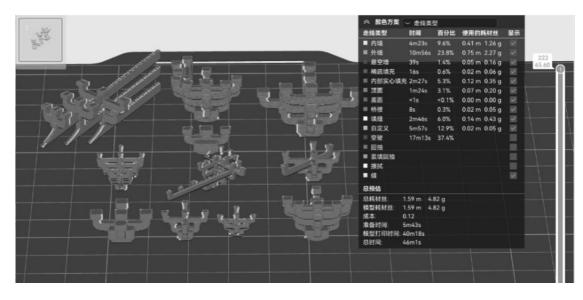


Fig. 2. Preliminary collation analysis of the model

curve, defined as shown in equation (4). In the figure, $\binom{n}{i}$ represents the binomial coefficient:

Using Bezier curves, the planning and calcula-

$$B_{i,n}(t) = \binom{n}{i} (1-t)^{n-i} t^i$$
(4)

tion of animation and motion trajectories in 3D printing ensure that the generated surface has good smoothness and flatness. This allows each adjacent curved surface to be accurately adjusted and refined. The obtained dougong model data is then reasonably extracted. During the extraction process, the following slicing formula is strictly applied, where N is the total number of layers, H is the total height, and h is the height of each layer, as shown in equation (5):

During the processing, the 3D model undergoes

$$N = \frac{H}{h}$$
(5)

voxelization, converting the model into a discrete voxel grid, which plays an important role in simulation, rendering, and printing on a computer. This process follows the formula (6):

In the equation, (x,y,z) represents the coordinates

$$V(x, y, z) = \begin{cases} 1 \text{ if } (x, y, z) \text{ is inside the object} \\ 0 & \text{otherwise} \end{cases}$$
(6)

of a voxel in 3D space, while V(x,y,z) determines whether the voxel is inside the target object. If it is inside, V(x,y,z)=1, otherwise it is 0. Voxelization

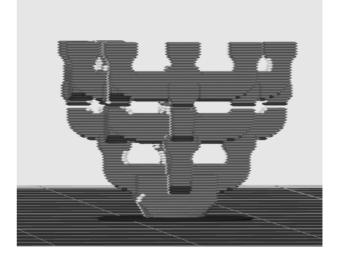


Fig. 3. Slice processing analysis

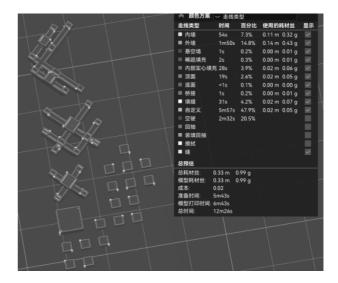


Fig. 4. Module splitting

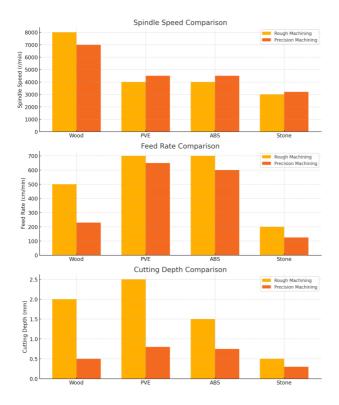


Fig. 5. Histogram comparison

facilitates the segmentation of complex geometric shapes and allows for efficient processing of these shapes $_{\circ}$

3.3. Jingdiao JDP fine chemical industrial production

3.3.1. Materials and Processing

In the design and processing of materials such as jade, wood, and PVC, fine carving technology plays an important role in detailed analysis and processing. Depending on the material, spindle speed, feed rate, and other parameters of Jingdiao data vary significantly. For the refinement of the dougong model created through 3D printing, the relevant parameters are presented in the table 1.

Table 1. Finishing parameters for different materials

3.3.2. Toolpath algorithm

The toolpath is an important parameter that needs to be accurately controlled in Jingdiao software, and the geometric planning of the route restores the authenticity of the original object. There are two modes of linear interpolation and circular interpolation, and circular interpolation is based on linear interpolation, so linear interpolation is the basic form, and its linear interpolation algorithm follows the following formula (7):

where P(t) is the position of the tool at time t, and P_1 and P_0 are the coordinates of the start and end points, respectively.

$$P(t) = P_0 + t(P_1 - P_0), 0 \le t \le 1$$
(7)

The algorithm for circular interpolation follows the following equation (8):

where (X_c, y_c) is the coordinate of the center of the circle, r is the radius of the arc, and $\theta(t)$ is the

$$P(t) = (x_c + r\cos(\theta(t)), y_c + r\sin(\theta(t)))$$
(8)

angle that changes with time t.

3.3.3. Tool compensation algorithms

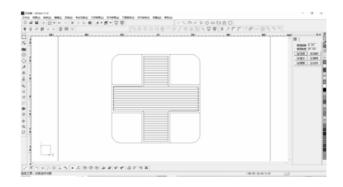


Fig. 6. Contour finishing

	Flat-bottomed bits (mm)	Rough machining			Fine machining		
Material		rotate speed (r/min)	Feed rate (cm/min)	The amount of the knife (mm)	rotate speed (r/min)	Feed rate (cm/min)	The amount of the knife (mm)
timber	10	8000	500	2	7000	230	0.5
PVE	2.5	4000	700	2.5	4500	650	0.8
ABS	3	4000	700	1.5	4500	600	0.75
Stone	15	3000	200	0.5	3200	125	0.3

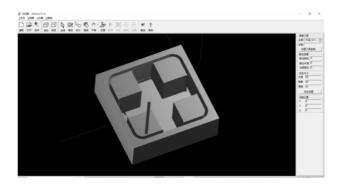


Fig. 7. Zone processing

In the actual machining, due to the shape number and size of the tool, the path trace of the multi-dimensional control does not conform to the walking path of the tool, so in the actual machining process, the physical properties of the tool will cause wear and error, so the tool compensation algorithm is used to correct the error of the tool radius or shape for the machining accuracy. The formula for the tool compensation algorithm is as follows (9):

where P_{corrected} is the corrected path, P_{path} is the

$$P_{\text{corrected}}(t) = P_{\text{path}}(t) + r\hat{n}(t)$$
(9)

original path, r is the tool radius, and $\hat{n}(t)$ is the normal of the path.

3.3.4. Production practices

To verify the feasibility of modular production, this paper focuses on mural sculpture, an integral part of ancient building construction. Using an Xray spectrometer to scan data for detailed processing, the workflow enables seamless data transfer between carving software, allowing the transformation from original object to model in an industrialized production process. Production trials are conducted on materials with varied properties to test the feasibility and sustainability of digital module production.

4. Results and Discussion

4.1. Strategies for digitizing the module chain

The in-depth exploration of the BIM+3D scanning module chain—from the preliminary design phase of a classical building to the completion of its final restoration—demonstrates the critical role of BIM+3D scanning technology. By utilizing BIM's advanced modeling tools, the collected data can be

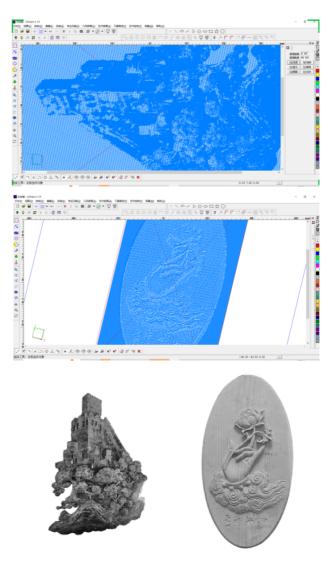


Fig. 8. Production practices

integrated and transformed into a detailed 3D model. This model clearly displays the various parts and decorations of the ancient building. With the assistance of this model, design experts and cultural heritage conservation staff can perform thorough analysis and develop more accurate, scientifically grounded conservation and restoration plans. For instance, during the restoration of an ancient building, it is crucial to precisely identify the size and shape of the damaged sections, allowing the selection of the most appropriate repair materials and techniques to ensure the restoration closely matches the building's original condition.

The integration of 3D printing technology with carving techniques offers more flexible possibilities for design approaches. The modularity of 3D printing combined with fine carving technology, which combines the advantages of rapid prototyping and detailed engraving, enables the creation of stunning quality, whether for complex industrial parts or high-end works of art. This not only advances The deep integration of mold-making technology and production methods has significantly improved industrial efficiency. By combining precise mold manufacturing techniques with efficient production processes, both product quality and production efficiency have been successfully enhanced. This integration benefits a wide range of industries, from artifacts to cultural products, driving progress and transformation across sectors.

The development of cultural creativity, when combined with its application in education, creates a strong foundation for both the inheritance and innovation of culture. On one hand, cultural and creative development incorporates cultural elements into various products, allowing culture to enter people's daily lives in a more engaging and enjoyable way. On the other hand, cultural and creative products serve as valuable teaching resources, enabling students to experience the allure of culture during the learning process while fostering their innovative thinking and cultural awareness.

4.2. innovation

This modular approach to thinking is bound to become more widespread in the future, serving as a key driver of social progress. The collaborative integration of multiple industries, fields, and directions will also receive unprecedented attention in terms of cultural dissemination and application. It not only facilitates resource sharing and the mutual enhancement of different industries but also creates a broader platform for the inheritance and innovation of cultural heritage.

The rapid expansion of digital technology will drive the deep integration of product functions, production technologies, business models, and consumption patterns across industries, shaping a new paradigm for industrial integration and development. Over the years, the government has consistently promoted policies supporting multi-industry integration to encourage and facilitate the development of "digital +" multi-industry initiatives. At the same time, as technology continues to evolve, new integration methods and application scenarios will gradually emerge. With the advancement of cuttingedge technologies such as artificial intelligence, big data, and the Internet of Things, broader opportunities for cross-industry integration will arise.

Overall, the multi-industry integrated development module chain of "digital +"—with its strong innovative capacity and vitality—provides a clear direction for future social progress. In this era of diverse opportunities and challenges, cross-industry integration is expected to play an increasingly critical role in the transmission and practice of culture, which represents its future development trend.

5. Conclusion

Thanks to the continuous advancement of digital software technology and the ongoing updates in scanning technology, the restoration, conservation, research, and development of cultural relics have been significantly accelerated. When working with a variety of complex data software, there is a tendency to modularize and consolidate numerous requirements. By adopting advanced sculpting and model-making technology that combines scanning with 3D printing, modular chain technology not only meets the needs of production activities but also adapts to the ongoing development trends of modern society. For cultural heritage to be more widely disseminated, the digital representation of cultural relics must be adjusted to transform objects that would otherwise be difficult to understand into easily comprehensible forms. Due to the diversity of design forms, the digital resources of cultural relics are more accessible during the process of sharing and dissemination, and can also meet the multiple needs of industrial manufacturing and media. This modular approach plays a decisive role in both preserving culture and ensuring its continuous progress.

With the introduction of national policies promoting the implementation of digital cultural technology, the development of the "digital +" model has become inevitable. This modular chain approach aligns with the mainstream development direction for the future and holds significant social, economic, and cultural value. The future of "digital +" will no longer be limited to a single industry, but will give rise to a "digital +" multi-industry integration model. Technologies such as BIM combined with 3D scanning, 3D printing coupled with carving, mold-making integrated with production modes, and cultural and creative development combined with teaching applications will play key roles. This modular chain concept will become increasingly popular in the future, with a focus on the dissemination and application of culture through multi-industry, multi-field, and multi-faceted collaboration and integration. Moving forward, the development of this digital technology module chain will continue to evolve, playing an even greater role in the transmission of culture.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal

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